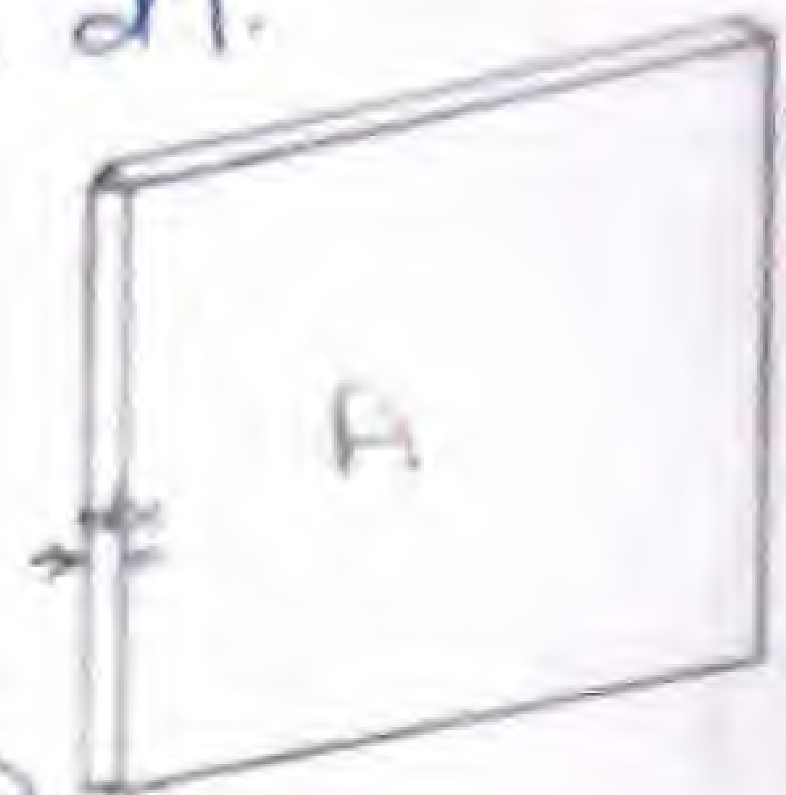
"Heat" chapter "1"

* Three methods of heat Transfer:
  conduction . Convection . Radiation.
* Thermal conductivity : $(H = Q/t)$

Codaction: The Transfer of heat through an unequally heat body from points of higher Temp. to points of lower Temp. without any motion of any part of the body.

* Proof:
  - Consider a thin slab of surface area A and thickness dx and Temp. difference between its faces dT.
  - The amount of heat energy flowing across slab per second dT.
  - The Rate of flow is found to be proportional to the Temp. difference gradient $\frac{dT}{dx}$ and also A.



$$H = Q/t \propto \frac{dT}{dx} A \Rightarrow H = K A \left(-\frac{dT}{dx}\right).$$

; K: "Thermal conductivity"

; "-ve sing": "$\frac{dT}{dx}$" is Rate of decrease of T with x.

Heat current: The amount of heat energy flowing across the slab perpendicular to the surface area A in 1"s".

Thermal conductivity coeffecient: The amount of heat energy flowing across the slab normal to the face of unit area A per unit time to cause unit Temp gradient.

* units of "k"

Joule / sec. m . °k

watt / m. °k.

cal. s^{-1} cm^{-1}. k^{-1}

1 calorie = 4.186 Joule.

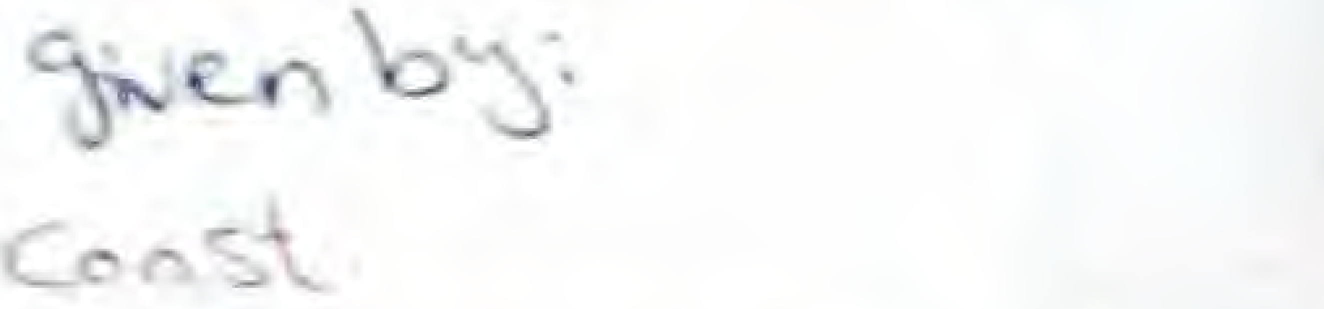
Materials have: → large K: Good conductors
→ less K: Bad conductors.

* steady flow of Heat
- This is very similar to steady flow of liquid and steady flow of electric charge
- In this case: Temp. doesn't change with time at any point of the body
- The resulting heat flow is steady flow and obeys the equation of continuity
- "T": const. with Time. Varing with position.

* Steady flow across a uniform rod
- consider the linear flow of heat which occurs along a uniform rod when no heat is allowed to leak.
- If rod is of length l and const. cross section A.
- Then the const. heat current is given by:

$$H = Q/t = -KA\frac{dT}{dx} = \text{Const}$$

- let us: Take one end of the rod as an origin and the axis-x along the axis of rod,



Hence, $dT = -\frac{H}{KA}\,dx$

$$\int dT = -\frac{H}{KA}\int dx \Rightarrow T = -\frac{H}{KA}x + \text{Const}$$

- suppose:

Temp is T_1 at $(x=0)$, T_2 at $(x=l)$.

- when: $T = T_1$ when $(x=0)$.

Hence, $T = \frac{-H}{KA}T + \text{Const} \Rightarrow T = \text{Const}$.

$T = T_1 - \frac{H}{KA}x \Rightarrow$ which determines the Temp. at any position x along the rod.

- The Heat current in the rod is determined by placing $(T = T_2)$ when $(x=l)$ So, $H = \frac{KA}{l}(T_1 - T_2)$, $T = T_1 - \frac{T_1 - T_2}{l}x$.

- Analog with ohm's law:

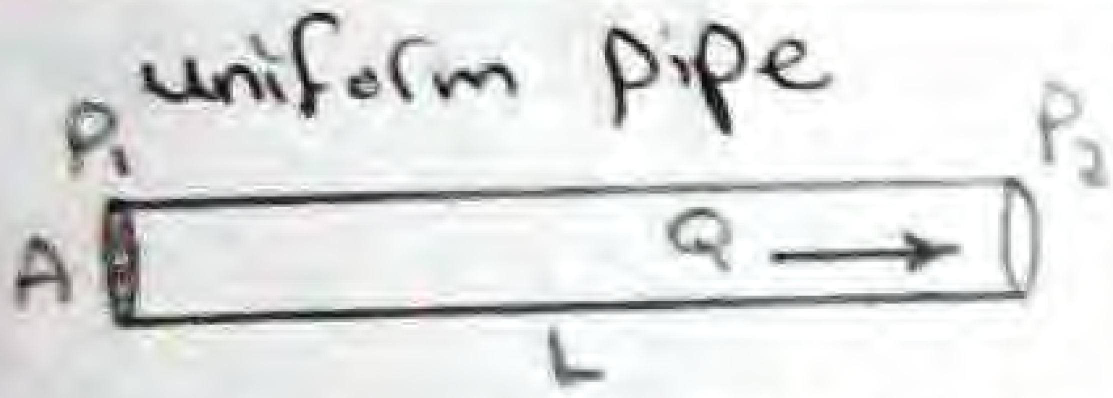
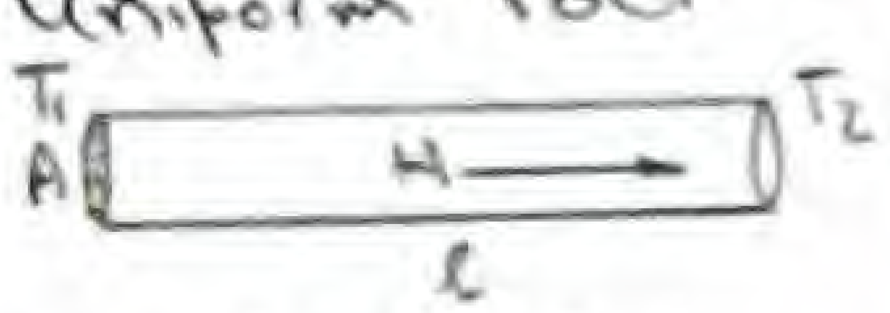
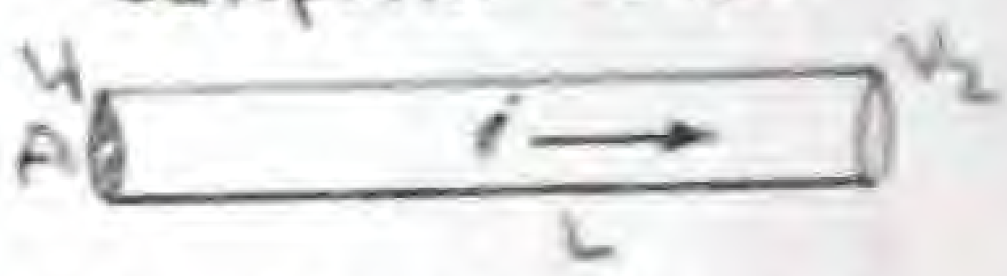
$$H = \frac{Q}{t} = \frac{(T_1 - T_2)}{l} KA.$$

- The equation is analogous to ohm's law for steady flow of electric charges in a metal conductor.

$$i = \frac{Q}{t} = \frac{V_1 - V_2}{R} = \frac{(V_1 - V_2)}{l} \sigma A.$$

$I = Q/t$ → electric current	$H = Q/t$ ⇒ Heat current
$(V_1 - V_2)$ → Potential difference	$(T_1 - T_2)$ → Temp. difference
σ ⇒ electric conductivity	K ⇒ Thermal conductivity
$(L/\sigma A)$ ⇒ electric Resistance "R"	(L/KA) ⇒ Thermal Resistance "R"

Analog between the laws governing the steady flow of "fluid, Heat, charges"

Stedy flow of fluids	--- of heat	...of electric charges
uniform pipe 	uniform rod 	uniform wire 
- By Poiseuille's equ.: $Q = \frac{\pi P a^4}{8\eta L} = \frac{P(\pi a^2)}{8\eta\pi \frac{L}{\pi a^2}}$ $Q = \frac{PA}{8\pi\eta \frac{L}{A}} = \frac{F}{R} \Rightarrow (1)$ ∴ $P = P_1 - P_2$ $F = P/A$	- By equation: $H = KA \frac{T_1 - T_2}{L} =$ $H = \frac{T_1 - T_2}{(L/KA)}$ $H = \frac{T_1 - T_2}{R} \Rightarrow (2)$	- By ohm's law: $i = \frac{V_1 - V_2}{f(L/A)}$ $i = \frac{V_1 - V_2}{(L/\sigma A)}$ $i = \frac{V_1 - V_2}{R} \Rightarrow (3)$

- Steady flow of a compound wall.

  - It is made of two slabs of different materials
  - connected by → Common surface (series connection)
    → Common edge (parallel connection)

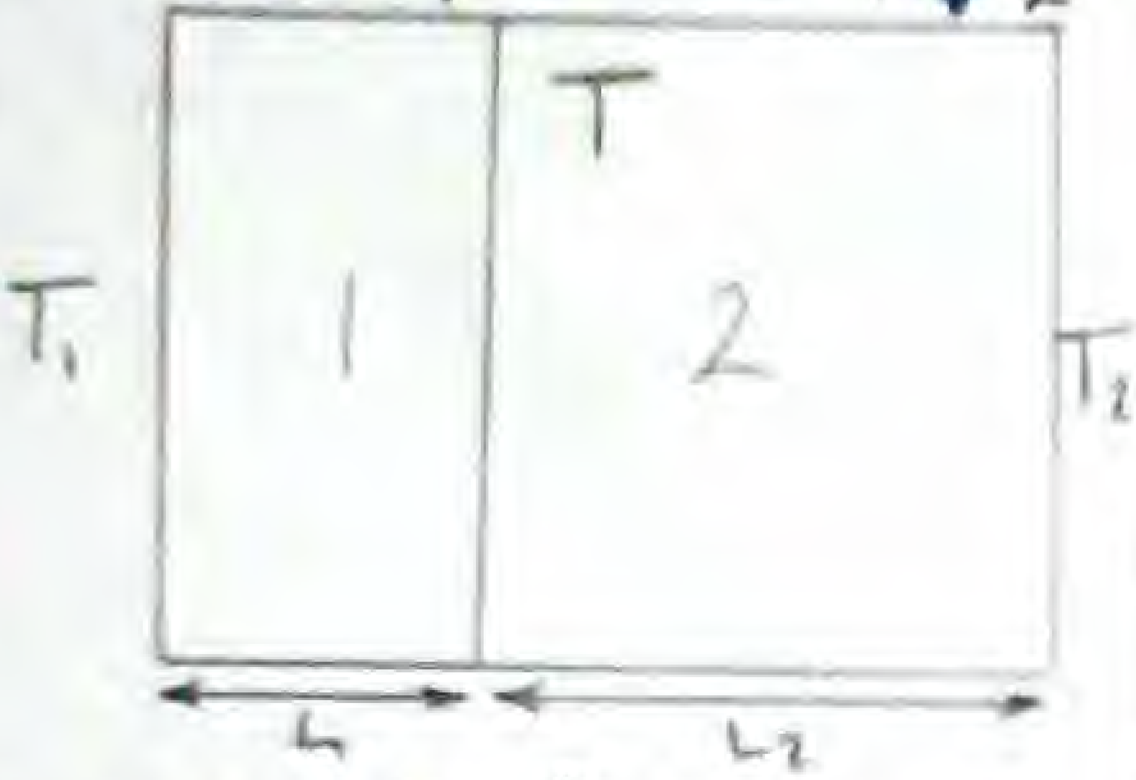
Series Connection

Consider compound wall made of two slabs "1" and "2".

- Thickness "L_1 , L_2"
- Thermal conductivity "K_1 , K_2"
- Outer surfaces of Temps "T_1, T_2" , "$T_1 > T_2$"

∵ we have steady flow.

∴ H is const. and equal in "1 & 2"



$$H = \frac{K_1 A}{L_1}(T_1 - T) = \frac{K_2 A}{L_2}(T - T_2)$$

- Putting: $\frac{K_1 A}{L_1} = \frac{1}{R_1}$ & $\frac{K_2 A}{L_2} = \frac{1}{R_2}$

$$H = \frac{1}{R_1}(T_1 - T) = \frac{1}{R_2}(T - T_2)$$

$$\therefore \frac{R_2}{R_1} = \frac{T - T_2}{T_1 - T}$$

$$R_2 T_1 - R_2 T = R_1 T - R_1 T_2$$

$$T(R_1 + R_2) = R_1 T_2 + R_2 T_1$$

$$\boxed{T = \frac{R_1 T_2 + R_2 T_1}{R_1 + R_2}}$$

$$T_1 - T = \frac{H}{K_1 A / L_1} , \quad T - T_2 = \frac{H}{K_2 A / L_2}$$

- By adding:

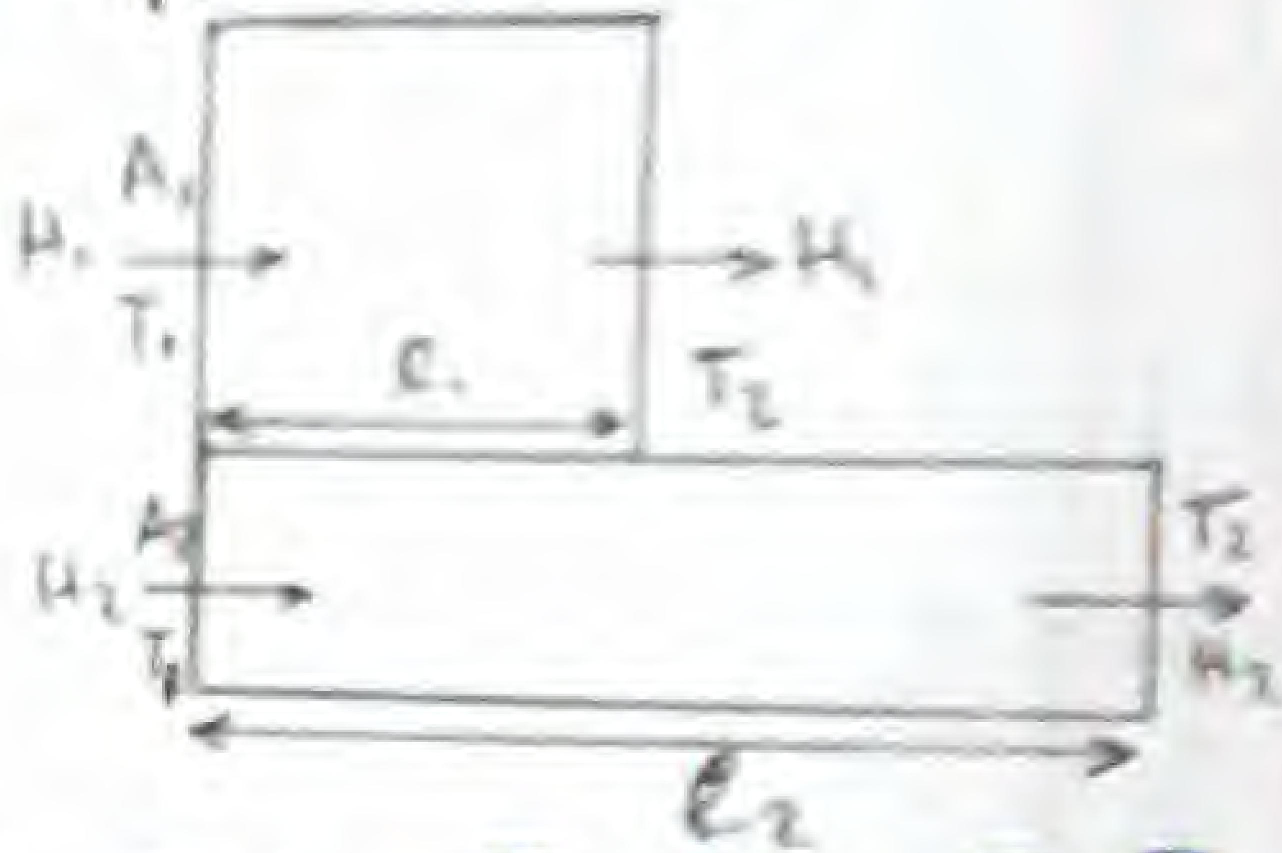
$$H\left[\frac{1}{K_1 A/L_1} + \frac{1}{K_2 A/L_2}\right] = T_1 - T_2$$

$$H \cdot [R_1 + R_2] = T_1 - T_2$$

$$\boxed{H = \frac{T_1 - T_2}{R_1 + R_2} = \frac{T_1 - T_2}{R}}$$

Parallel connection

- Consider two slabs "1 & 2"
- Area "A_1 , A_2"
- Thermal conductivity "K_1 , K_2"
- attached to form a wall.
- Temp. of surface of both slabs. "T_1 & T_2"
- Heat Current is equal to The sum: "$H = H_1 + H_2$"



$$H_1 = \frac{K_1 A_1}{L_1}(T_1 - T_2) = \frac{T_1 - T_2}{L_1 / K_1 A_1} = \frac{T_1 - T_2}{R_1}$$

Simillarly: $H_2 = \frac{T_1 - T_2}{R_2}$

Then: $H = H_1 + H_2$

$$H = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$\left(\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}\right)$: Thermal Resistance

$$\left(H = \frac{T_1 - T_2}{R}\right)$$

$$\boxed{H = \frac{T_1 - T_2}{R}}$$

- This is analogous to the parallel connection of electrical resistance in electricity.

- Consider a hollow sphere of inner radius r_1 and outer radius r_2.

- Inner surface at Temp. T_1 and outer at Temp T_2 "$T_1 > T_2$"
- let the coefficient of thermal conductivity "K"
- Heat flows outwards from the inner to the outer surface.
- $\therefore$ the flow is steady. So H is the same across any shell
- Consider a spherical shell of radius r and thickness dr.

$$\boxed{H = -KA\frac{dT}{dr}} \; ; \; dT/dr : \text{Temp. gradient across shell}$$

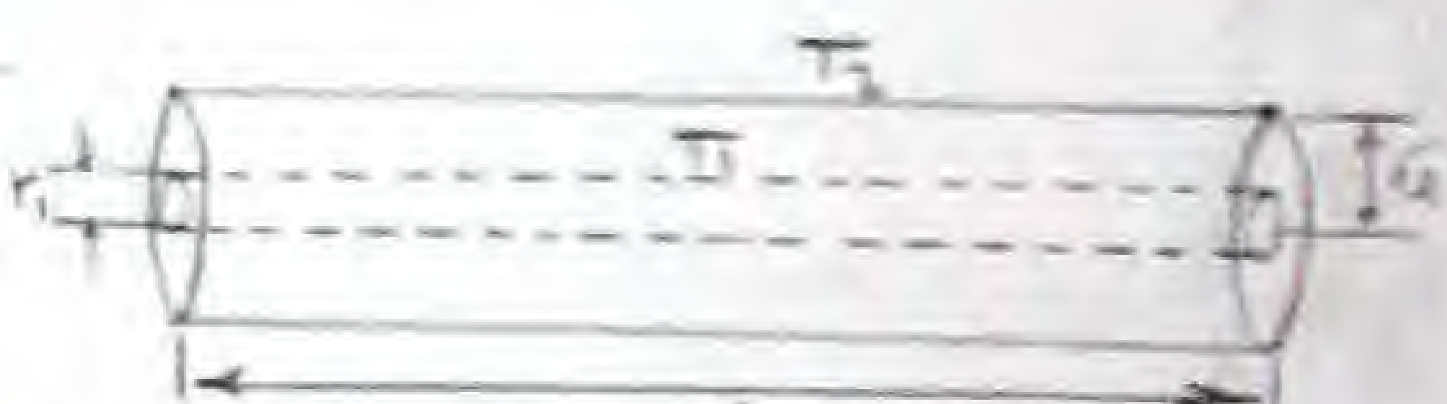
$$\therefore A = 4\pi r^2 \qquad \therefore H = -K(4\pi r^2) \cdot \frac{dT}{dr}$$

$$H\int_{r_1}^{r_2}\frac{dr}{r^2} = -4\pi K\int_{T_1}^{T_2} dT \Rightarrow -H\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = -4\pi K(T_1 - T_2)$$

$$H\left(\frac{r_2 - r_1}{r_1 r_2}\right) = 4\pi K(T_1 - T_2)$$

$$\therefore \boxed{H = 4\pi K(T_1 - T_2)\left(\frac{r_1 r_2}{r_2 - r_1}\right)}$$

- steady Heat flow through a cylinderical wall:
- Consider a cylinder of Radius r
- thickness dr and length ℓ.



$\therefore$ flow is stedy.

$\therefore$ H is the same across any cylinder.

$$H = -KA\frac{dT}{dr} \quad ; \quad A = 2\pi r\ell.$$

$$H = -K(2\pi r\ell)\frac{dT}{dr} \Rightarrow H\int_{r_1}^{r_2}\frac{dr}{r} = -2\pi K\ell\int_{T_1}^{T_2} dT$$

$$H \cdot (\ln r_2 - \ln r_1) = -2\pi K\ell(T_2 - T_1)$$

$$H \cdot \ln\left(\frac{r_2}{r_1}\right) = -2\pi K\ell(T_2 - T_1)$$

$$\boxed{H = 2\pi K\ell\frac{(T_1 - T_2)}{\ln(r_2/r_1)}}$$

Convection

- When mass of liquid is heated its volume increases so the density decreases.
- If the liquid is in a beaker which is heated by a flame beneath it, the hot liquid rises to the top and the cold falls to bottom where it becomes hot in turn and rises.
- In this way the liquid becomes hotter.

definition: It is the transfer of heat from one place to another by the movement of the substance itself between these places.

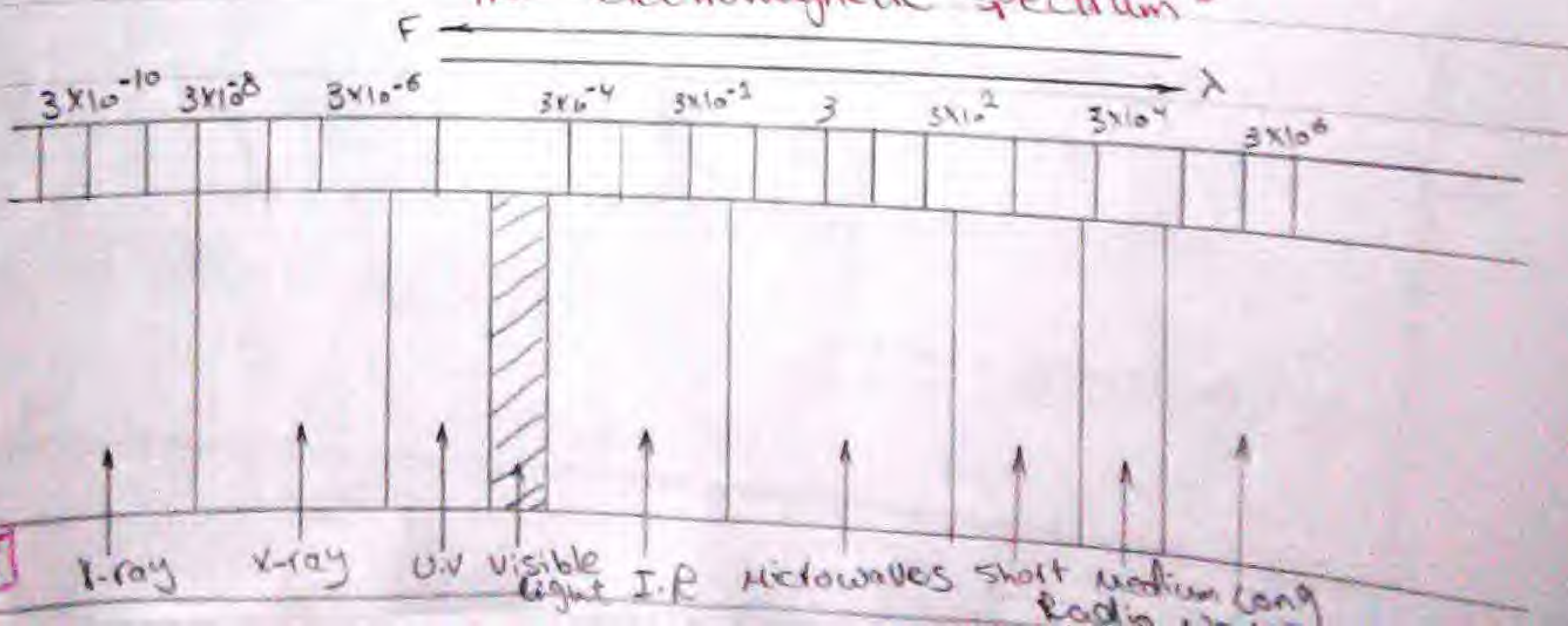
- Convection occurs only in liquid and gas not solid.

Radiation

- It is the transfer of heat energy from a hot body to a cooler body without appreciable heating of the intervening space.
- Transverse electromagnetic waves ($V = 3 \times 10^8$ m/S)
- The characteristic properties of the different types of waves are due to difference in frequency or wavelength

$\boxed{\lambda = \frac{c}{F}}$ $\boxed{\lambda_{max} . T = 0.003 \text{ m.K}}$

"The electromagnetic spectrum"

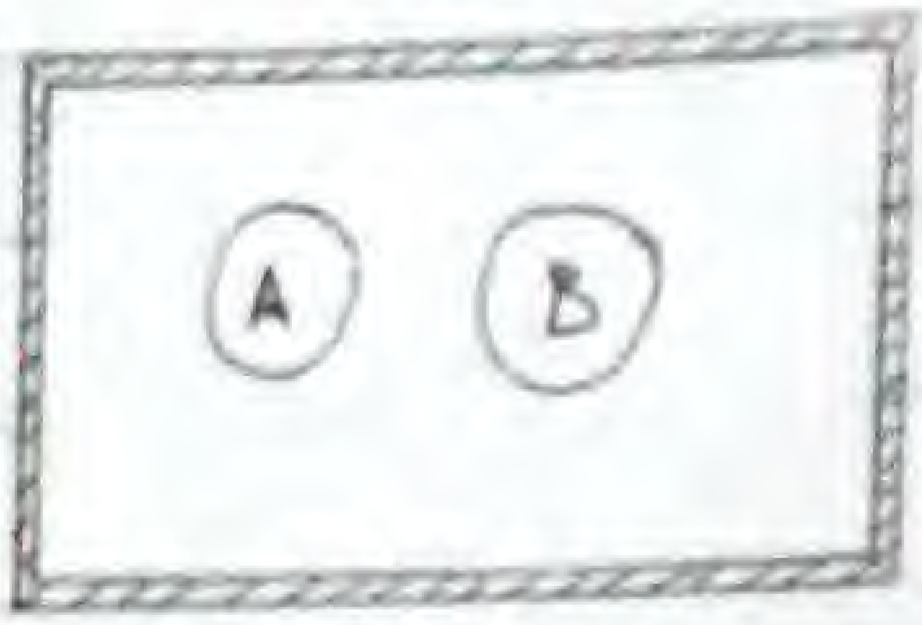


- Theory of exchanges

This theory states that " All bodies are continually radiating thermal energy, and at the same time absorbing it from their surroundings".

- Consider :

- Non conducting evacuated enclosure.
- Walls at a definite Temp.
- Body A " Temp of A > Temp of wall"
- Body B " Temp of B < Temp of wall"



→ Experiment shows that : " A becomes colder, B becomes warmer and Temp. of walls changes until all of A, B and walls are the same Temp.".

→ The final state called " Thermal equlibrium"

Note: - AT "Thermail equilibrium" radiation doesn't stop but "The energy absorbed = energy Radiated"

- Emissive power "Emissivity"

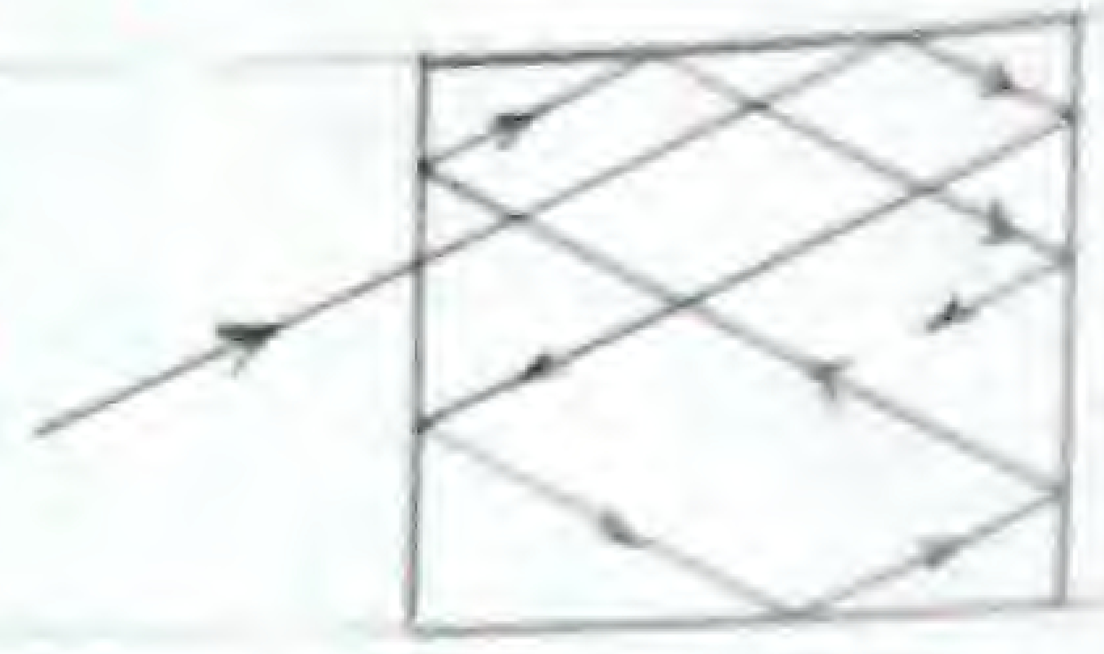
- Emissivity : The rate at which thermal radiation is lost from the surface of the body.

depends on: - The nature of the surface.
- The difference between its Temp and surr.
- The material which consititutes the body

- Emissive power : The radiant energy emitted per unit time from a unit area of the surface.

Note: - It is denoted by "e"
- "e" is a function of the wave length of the emitted Radiations.

- Absorptivity and Reflectivity
  - let Fraction a is absorbed $\xrightarrow{OR}$ absorptivity.
  - fraction ρ is reflected $\xrightarrow{OR}$ reflectivity
- opaque body:
  - Radiation transmitted = 0.
  - Then, $a + \rho = 1$.
- Black body:



- Any radiation striking the hole enters the cavity where it is trapped by reflection until it absorbed
- The cavity walls constantly emitting and absorbing radiation
- ∴ The Radiation inside the cavity is black body radiation

Note Incase of Black.body , $a = 1$.

- Kirchhoff's law. "$e/a = e_b$ = Const"
- Definition: The ration of the emissive power to the absorptivity is the same for all bodies at the same Temp. and is equal to the emissive power of a black body at this Temp.

proof: - dQ: radient energy incident on unit area per unit time of a body of surface area A
- $A\,a\,dQ$: The energy absorbed by the body per "t".
- Ae: The radiation energy which surface emits in same "t"
- If Temp. remains Const.
- Then: $Ae = A\,a\,dQ \xrightarrow{OR} e/a = dQ$ — (1)
- For black body: $a = 1 \longrightarrow \therefore e_b = dQ$ — (2)

By (1) and (2):

$$\boxed{e/a = e_b = \text{Const}}$$

Stefan Boltzmann's law

definition: The emissive power of a black body is proportional to the fourth power of its absolute Temp.

$$\boxed{e_b = \sigma T^4}$$

σ: stefan Boltz const

σ = 5.678×10^{-8} Joule/(m^2.sec°K^4)

special case

- The thermal Radiation from many real surfaces which are not black is found to be nearly proportional to "T^4" but with a proportionality const is smaller than "σ"

$\therefore e_b = e/a \quad \therefore \boxed{e/a = \sigma T^4}$

- Net loss or Heat by Radiation
- consider a body and its surrounding of Temp. "T_0"

$\therefore$ Rate of absorption = Rate of emission

$\therefore H_1 = A e_0 = A a \sigma T_0^4$

- **let**: The Temp of body "T" as $T > T_0$
- "The Rate of absorption and Temp of surrounding" are const

$\therefore$ The rate of emission: $H_2 = Ae = A a \sigma T^4$.

- The net loss of radiant energy is:

$$\boxed{\begin{aligned} H &= H_2 - H_1 = A a \sigma (T^4 - T_0^4) \\ H &= \text{const } (T^4 - T_0^4) \end{aligned}}$$

-

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- Newton's law of cooling

- when The Temp difference between the body and its surrounding is not too large, the rate of change of Temp of the body is approximately proportional to the difference between its Temp and that of its surrounding.

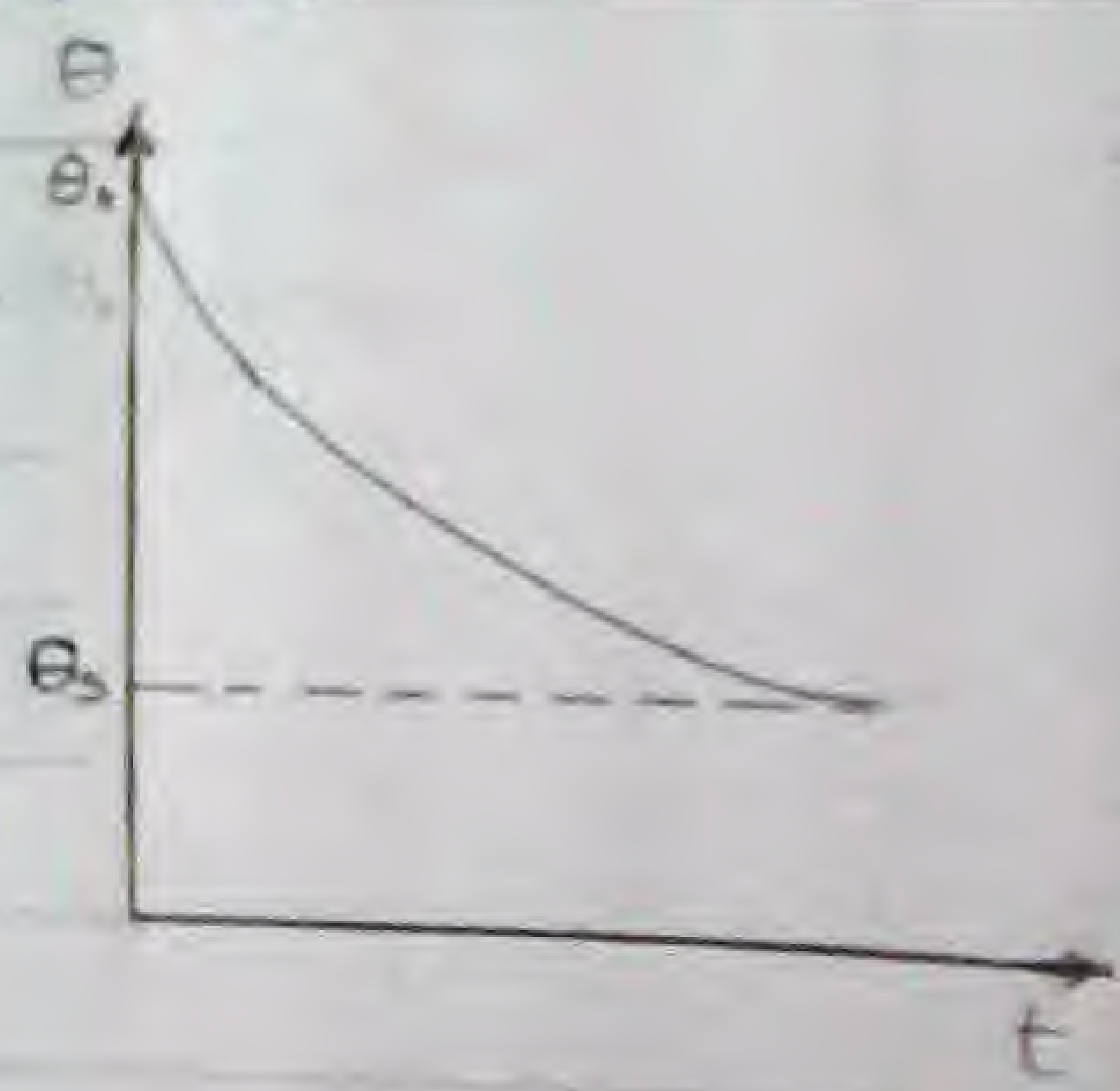
- θ : The Temp of a body
  θ_s : The Temp. of surrounding

Then, $\frac{d\theta}{dt} = -k(\theta - \theta_s)$.

- k : Const
- The negative sign : when Time "t" increases "θ" decreases

$$\frac{d\theta}{\theta - \theta_s} = -k\,dt \xrightarrow[\text{both sides}]{\text{Integrating}} \ln(\theta - \theta_s) = -kt + C.$$

If : θ_0 is Temp of body when $t = 0$,



$\therefore C = \ln(\theta_0 - \theta_s)$

$\therefore \ln(\theta - \theta_s) = -kt + \ln(\theta_0 - \theta_s)$

$\therefore \ln\left(\frac{\theta - \theta_s}{\theta_0 - \theta_s}\right) = -kt$

$\therefore \boxed{\theta = \theta_s + (\theta_0 - \theta_s)e^{-kt}}$.

chapter "1" Heat
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